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Nonlinear Interactions in a Piezoceramic Bar Transducer Powered by a Vacuum Tube Generated by a Nonideal Source

Interactions between the oscillations of piezoceramic transducer and the mechanism of its excitation—the generator of the electric current of limited power-supply—are analyzed in this paper. In practical situations, the dynamics of the forcing function on a vibrating system cannot be considered as given a priori, and it must be taken as a consequence of the dynamics of the whole system. In other words, the forcing source has limited power, as that provided by a dc motor for an example, and thus its own dynamics is influenced by that of the vibrating system being forced. This increases the number of degrees of freedom of the problem, and it is called a nonideal problem. In this work, we present certain phenomena as Sommerfeld effect, jump, saturation, and stability, through the influences of the parameters of the governing equations motion.
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Introduction

Usually, an experimental structural analysis supposes that the excitation devices are ideal or, in other words, nondependent of the dynamic response of that structure. Electric motors are an example of nonideal excitation devices because the mechanical output power depends on the motion of its armature and on the dynamic characteristics of its rotor, among other factors.

This kind of the nonideal problem was described in the classical book of Kononenko [1]. Nayfeh and Mook [2] gave a comprehensive and complete review of different approaches to the problem up to 1979. Recently, complete reviews of this kind of nonideal vibrations can be found in Balthazar and co-workers [3,4] and some perturbation methods for nonideal system was investigated as some examples, without undeserved others, we mention that Palacios et al. [5] used the method of averaging and Bolla et al. [6] used the method of multiple scales.

We also remarked that when motors are attached to structures that need excitation power levels similar to the power capacity of those motors, interesting nonlinear phenomena may happen, such as modal saturation and the Sommerfeld effect.

The experimental research detected the Sommerfeld effect; as the motor accelerates to reach near resonant conditions, a considerable part of its output energy is consumed to generate large amplitude motions of the structure and not to increase its own angular speed. For certain parameters of the system, the motor can get stuck at resonance not having enough power to reach higher rotation regimes. If some more power is available, the jump phenomena may occur from near resonance to considerably higher motor speed regimes, no stable motions being possible between these two. Recently, Dantas and Balthazar [7] presented the Sommerfeld effect as a bifurcation of periodic orbits, using the regular perturbation theory.

The saturation phenomenon was presented in Refs. [8,9], where we present measurements of the nonlinear oscillations of a portal frame foundation for a nonideal motor. We considered a three-time redundant structure with two columns, clamped in their bases and a horizontal beam. An electrical unbalanced motor is mounted at midspan of the beam. Two nonlinear phenomena are studied: (a) mode saturation and energy transfer between modes, and (b) interaction between high amplitude motions of the structure and the rotation regime of a real limited power motor. The dynamic characteristics of the structure were chosen to have one-to-two internal resonance between the antisymmetrical mode (sway motions) and the first symmetrical mode natural frequencies. As the excitation frequency reaches near resonance conditions with the second natural frequency, the amplitude of this mode grows up to a certain level and then it saturates. The surplus energy pumped into the system is transferred to the sway mode, which experiences a sudden increase in its amplitude. Energy is transformed from low amplitude high frequency motion into high amplitude low frequency motion. Such a transformation is potentially dangerous.

In this paper, we deal with an interesting nonideal problem, which was considered before by Krasnopolkaya and Shevts [10].

Here, we analyzed the phenomenon of saturation and of stability using averaging method, extending this previous paper. The subject of the present research is related to the real fact that the function of many important and mission-critical devices of various engineering machines, used in modern rocket technology, submarine devices, and transformers, is based on the effect of the coupling of mechanical and electrical fields in piezoceramic mediums [11,12].

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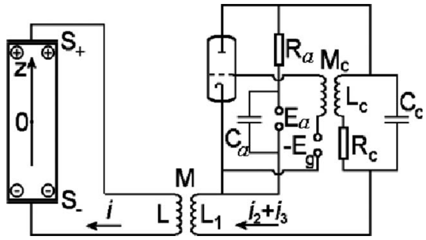

Fig. 1 Scheme of the nonideal piezoceramic rod transducer (left circuit) and vacuum-tube generator (right circuit) system

The general mathematical theory of electroelastic processes in such mediums under arbitrary conditions of mechanical and electrical loading is very important, both in scientific and applied aspects. However, in these theories, and in other publications, a problem of behavior of electroelastic fields is considered only for conditions of forced and free oscillations, when the piezoelectric ceramics is under activity of applied mechanical and electrical fields of given values. Thus, a problem of influence of dissipation and radiation of energy under oscillations of coupled fields of the device remains outside of many considerations. If the transducer with electroelastic coupled field is mounted in medium with resistance, as what happens in operation of sound emitters, then the radiation of energy changes the electric field in the power generator, as opposed to the "ideal" case when losses do not happen. This adjustment can be essential and lead to unexpected dynamic conditions or be negligibly small; it depends on the outer power of the generator compared with an emitting power. Examination of new effects in the dynamics of piezoceramic coupled fields and in the function of the power generator, which are caused by "sensitivity" of cumulative systems to radiation of energy, without a doubt present significant scientific interest. It is a case of so-called limited or nonideal excitation, when supply power is of the same order as consumed by a loaded piezoceramic transducer. In this case, the electric generator is said to have limited power, i.e., power comparable with the power radiated or consumed by piezoceramic coupled field.

The present paper is devoted to the analysis of interaction effects, in the oscillations of piezoceramic transducer and in the mechanism of its excitation—the generator of the electric current of limited power-supply. The new mathematical model of interaction of the generator and the piezoceramic transducer submerged in a hydromedium with resistance is constructed.

The coupling of processes in the transformer and the energy source (the generator) leads to such qualitatively new effects in their dynamics as cannot be seen using a model of the problem with unlimited or so-called "ideal" excitation—primarily the possibility of appearance of deterministic chaotic regimes, which are theoretically impossible in a problem with ideal excitation (when the corresponding mathematical models of such a problem have dimensionality of phase spaces equal to two, a possibility of chaos origination is excluded)—(see, for details, Ref. [10]). Furthermore, we will analyze the phenomenon of saturation and of stability using the averaging method, extending the previous one by Krasnopolkaya and Shevts [10].

Modelling of a Generator-Transducer System

In this section, we considerer a piezoceramic bar transducer (Fig. 1), which is embedded on the acoustical medium and whose electrodes are under applied electric voltage from a LC circuit generator. The presence of energy dissipation changed the parameters of steady state the generator and transducer. Note that the origin of the Cartesian coordinate system is the middle of the bar between its extremities S_- and S_+ that are perpendicular to axis OZ, acoustic signals radiate through the medium, i and L are the current and the capacitor of the transformer with the bar. i_2 and i_3 are the currents, R_a and R_c are the resistors, L_c and L_1 are the inductors, and C_a and C_c are the condensers of the generator. E_g and E_a are the constant component of the tube grid voltage and anodic voltage, respectively. M and M_c are the magnetic fields. This paper deals with the nonlinear dynamic analysis of the longitudinal vibrations of a round bar of length $2h$ and cross-sectional area S, with longitudinal polarization.

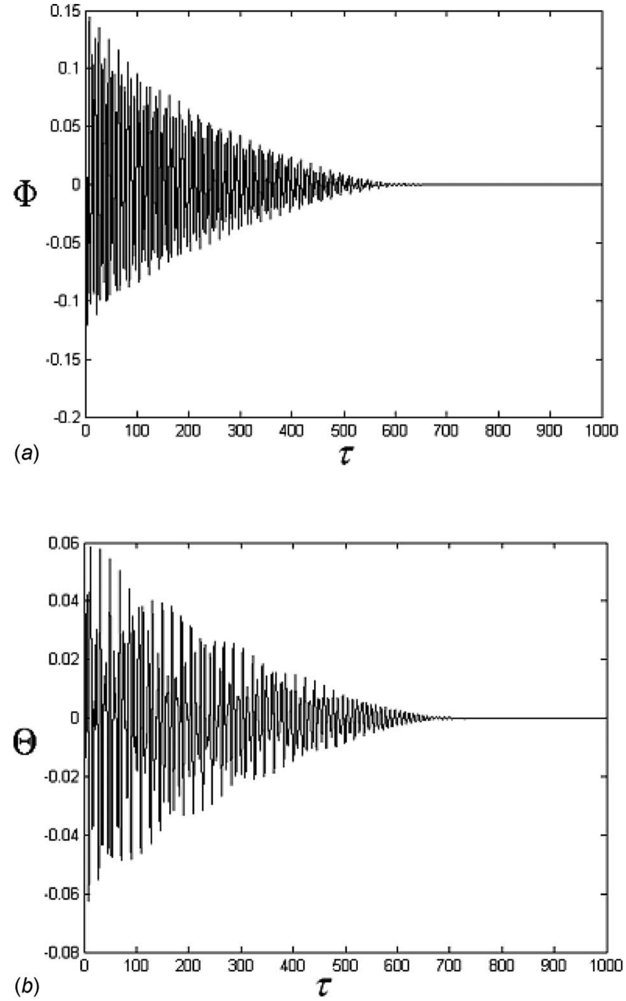

Fig. 2 Time response for $\alpha_0=0.5$ and $\alpha_4=0.23$: (*a*) generator, (*b*) transducer

The system of equations were obtained by Krasnopolskaya and Shevts [10] and they may be rewritten in dimensionless state variables form, as follows:

$$
\begin{aligned}
u_1' &= u_2 \\
u_2' &= -u_1 + \alpha_1 u_2 + \alpha_2 u_2^2 - \alpha_3 u_2^3 - \alpha_4 u_3 \\
u_3' &= u_4 \\
u_4' &= -\alpha_0 u_3 + \alpha_5 u_1 + \alpha_6 u_2 - \alpha_7 u_4
\end{aligned}
\tag{1}
$$

where $u_1=\Phi$ represents the displacement of the generator, $u_2=\Phi'$, $u_3=\Theta$ represents the voltage in the electrodes of the piezoceramic rod transducer, and $u_4=\Theta'$.

For determining the Jacobian matrix, we introduce the vector notation $\boldsymbol{u}'=\boldsymbol{U}(\boldsymbol{u})$ for the system (1) where $\boldsymbol{u}=[u_1,u_2,u_3,u_4]^T$, $\boldsymbol{U}=[U_1,U_2,U_3,U_4]^T$, $U_1=u_2$, $U_2=-u_1+\alpha_1u_2+\alpha_2u_2^2-\alpha_3u_2^3-\alpha_4u_3$, $U_3=u_4$, $U_4=-\alpha_0u_3+\alpha_5u_1+\alpha_6u_2-\alpha_7u_4$, and $[\cdots]^T$ denotes transpose.

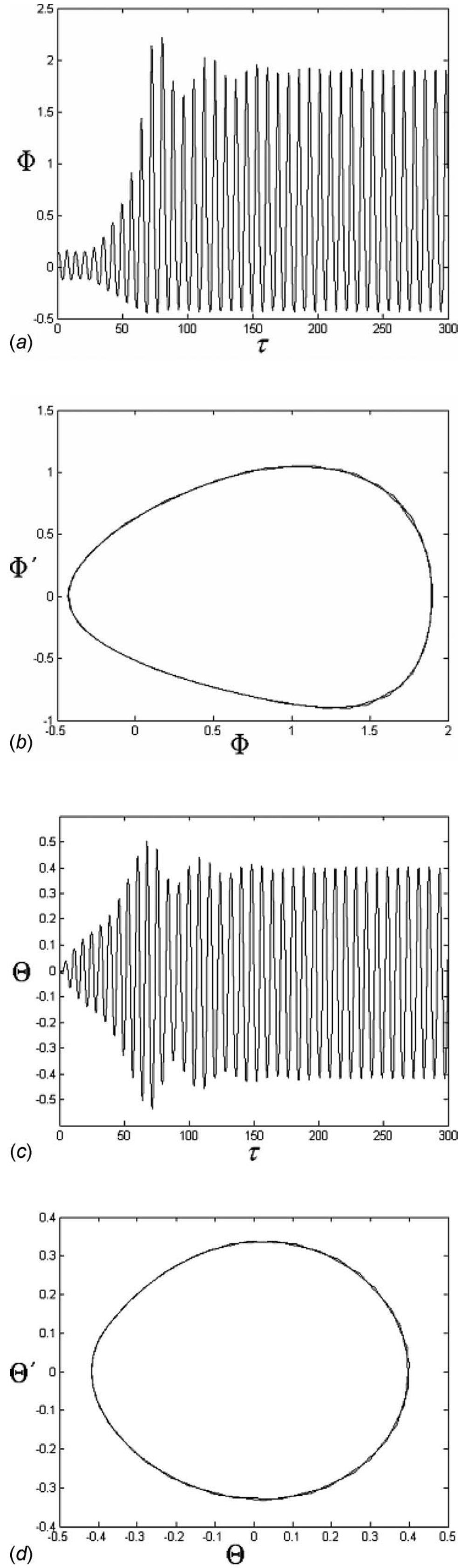

Fig. 3 Numerical result for α_0=0.9 and α_4=0.23: (*a*) and (*b*) generator response, (*c*) and (*d*) transducer response

The Jacobian matrix, defined by $J=[J_{ij}]=[\partial U_i/\partial u_j|_{\mathbf{0}}]$ in $\mathbf{0}=(0,0,0,0)$ is

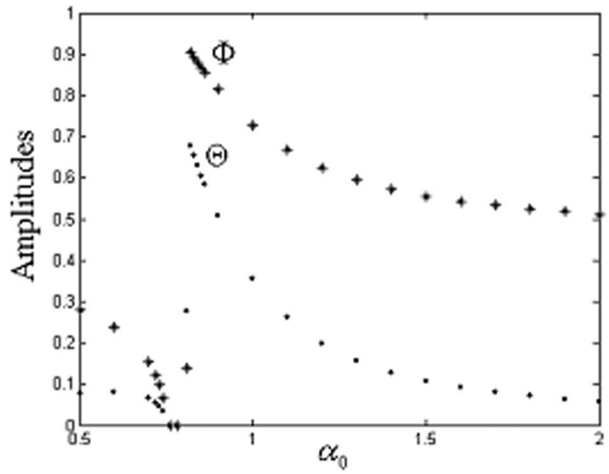

Fig. 4 Amplitudes-parameter curves

$$J=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & \alpha_1 & -\alpha_4 & 0 \\ 0 & 0 & 0 & 1 \\ \alpha_5 & \alpha_6 & -\alpha_0 & -\alpha_7 \end{bmatrix} \tag{2}$$

The characteristic equation associated with the matrix J

$$\lambda^4+(\alpha_7-\alpha_1)\lambda^3+(1+\alpha_0-\alpha_1\alpha_7)\lambda^2+(\alpha_7+\alpha_4\alpha_6-\alpha_0\alpha_1)\lambda + (\alpha_0+\alpha_4\alpha_5)=0 \tag{3}$$

and according to the criterion of Hurwitz, sufficient conditions for asymptotic stability of the equilibrium solution can be written in the following form:

$$\begin{gathered} \alpha_7-\alpha_1>0 \\ 1+\alpha_0-\alpha_1\alpha_7>0 \\ \alpha_7+\alpha_4\alpha_6-\alpha_0\alpha_1>0 \\ \alpha_0+\alpha_4\alpha_5>0 \\ (\alpha_7-\alpha_1)(1+\alpha_0-\alpha_1\alpha_7)-(\alpha_7+\alpha_4\alpha_6-\alpha_0\alpha_1)>0 \\ (\alpha_7+\alpha_4\alpha_6-\alpha_0\alpha_1)(1+\alpha_0-\alpha_1\alpha_7)(\alpha_7-\alpha_1)-(\alpha_7+\alpha_4\alpha_6-\alpha_0\alpha_1)^2 - (\alpha_0+\alpha_4\alpha_5)(\alpha_7-\alpha_1)^2>0 \end{gathered} \tag{4}$$

Choosing a suitable values for the parameters, we obtain the stability: $0.482\le\alpha_0\le 0.542$, $0.222\le\alpha_4\le 0.251$ and $0.056\le\alpha_7\le 0.29$.

We remark that we choose the dimensionless parameters as

$$\begin{gathered} \alpha_3=0.21\times X,\quad \alpha_4=0.103,\quad \alpha_5=-0.0604,\quad \alpha_6=-0.12 \\ \alpha_7=0.06,\quad \alpha_2=0.62\times X,\quad \alpha_1=0.0535,\quad X=2 \end{gathered} \tag{5}$$

and the initial conditions as $\Phi(0)=\Phi'(0)=0.1$, $\Theta(0)=\Theta'(0)=0$, in order to carry out the numerical simulations.

Here, α_0 is the parameter that relates ω_1 (the natural frequency of the piezoceramic transducer) to ω_0 (the natural frequency of the generator) of the form $\alpha_0=(\omega_1^2/\omega_0^2)$, where they are defined by

$$\omega_1=\sqrt{\frac{2h}{LS\varepsilon_{33}(1-k^2)}},\quad \omega_0=\sqrt{\frac{R_a+R_c}{R_aL_cC_c}} \tag{6}$$

where $k=d_{33}(\varepsilon_{33}s_{33})^{1/2}$, ε_{33}, d_{33}, and s_{33} are constants of the piezo-effect constitutive relations, according to the theory of longitudinal deformations: $\varepsilon_z=s_{33}\sigma_z+d_{33}E_z$, $D_z=\varepsilon_{33}E_z+d_{33}\sigma_z$, where ε_z is a longitudinal deformation, σ_z is the mechanical stress, E_z is

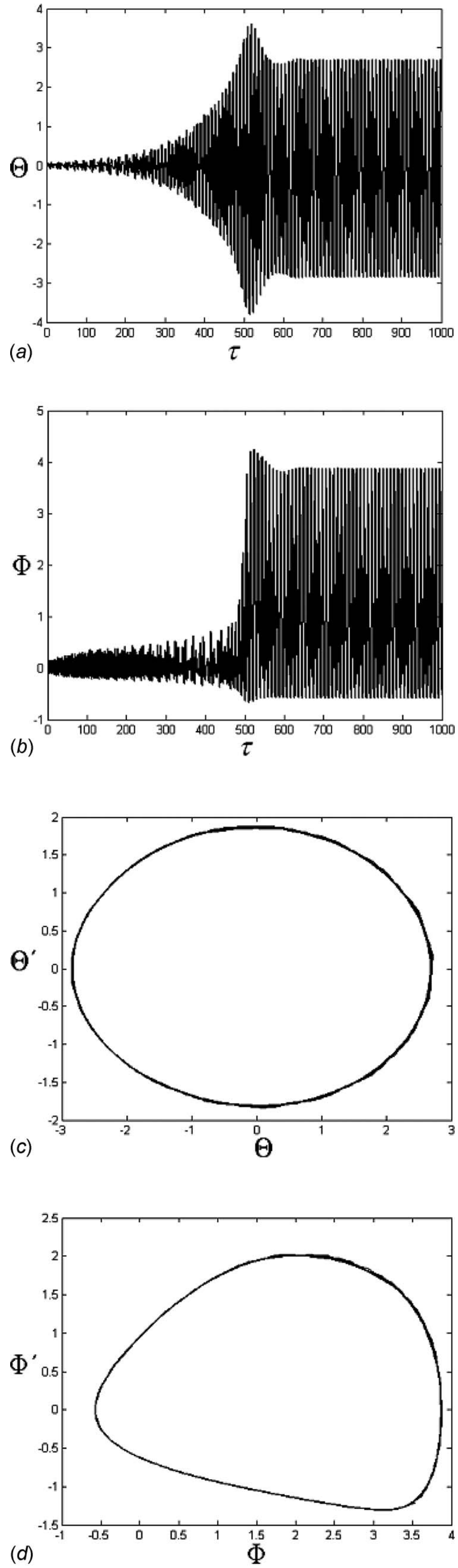

Fig. 5 (*a*) and (*b*) Time histories; (*c*) and (*d*) phase planes for $\alpha_0=0.5$ and $\alpha_4=0.103$

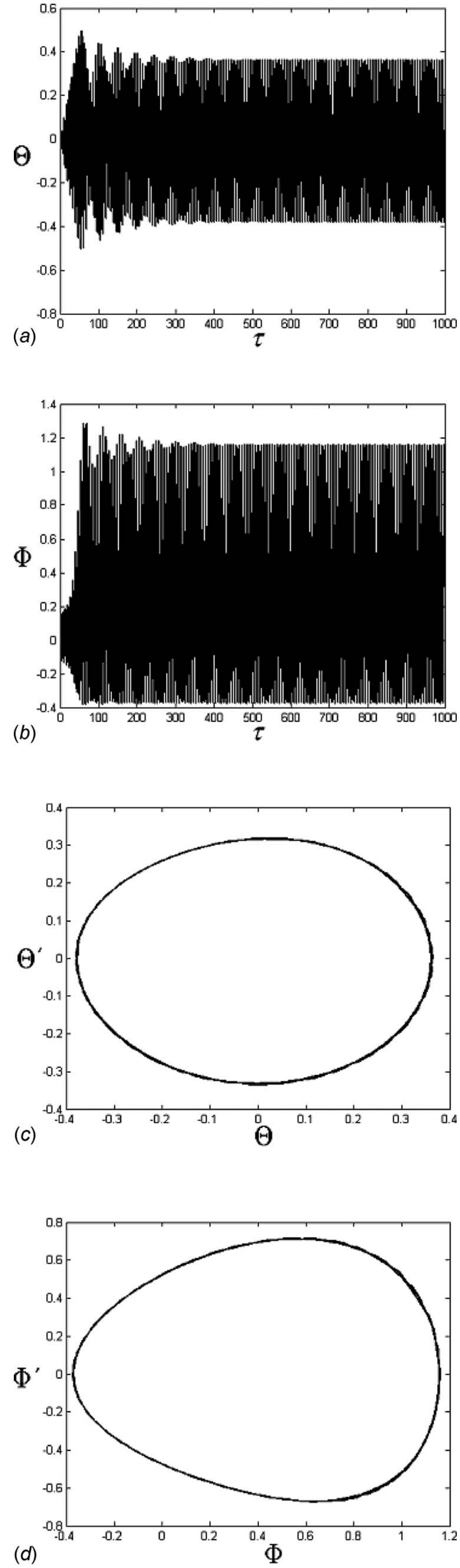

Fig. 6 (*a*) and (*b*) Time histories; (*c*) and (*d*) phase planes for $\alpha_0=1.0$ and $\alpha_4=0.103$

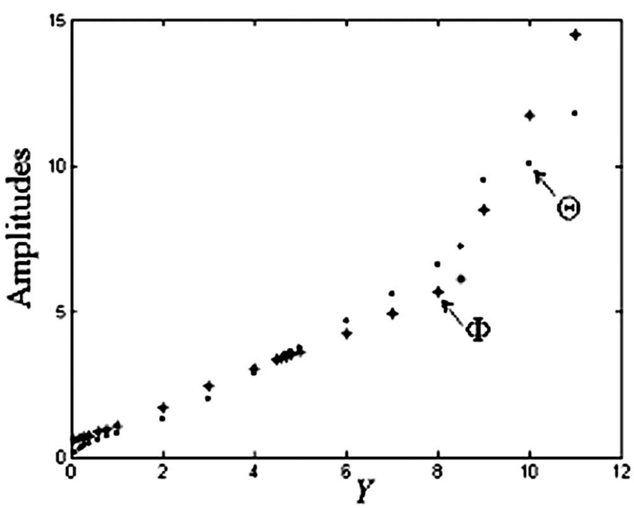

Fig. 7 Results of the nonideal and saturation phenomenon

the intensity of the electric field, and D_z is an induction of this field; R_a, R_c, L_c, and C_c are the resistors, inductor, and condenser, respectively, of the generator [10].

Figure 2 shows the numerical simulations results of stability of the system when $\alpha_0=0.5$ and $\alpha_4=0.23$. Figure 3 shows the instability of system when $\alpha_0=0.9$ and the steady state is periodic.

Next, we consider α_0 as a parameter of bifurcation in the numerical simulation those were carried out. According to the Eq. (6) this control parameter will describe the influence of the coefficients mechanical stress and intensity of the electric field [4]. In this case we may observe the influence of the internal resonance between ω_1 and ω_0.

In Fig. 4, we show the jump phenomenon of the amplitudes of the transducer-generator system when $\alpha_0 \in [0.5, 2.0]$ and the influence of the transducer vibrations (electric and mechanical) on the operation of the generator, in the steady state regime. In the value of $\alpha_0=0.81$ (unstable) exists an energy transfer between the generator response Φ and transducer response Θ with characteristics of the phenomenon of saturation.

In Figs. 5 and 6, we show the time histories and phase portrait plane, for two values of the α_0 parameter. Figure 5 shows the Sommerfeld effect, the left and right traces represent the response of transducer Θ and generator Φ, respectively. It can be clearly seen that the vibration amplitude of Θ is increasing, while the oscillations of Φ has small amplitudes until a certain time and results. That, it suffers a jump of oscillations of great amplitudes.

Figure 6, shows the absence of the effect of Sommerfeld.

Next, the parameter Y is elected as the bifurcation one. Figure 7 shows the effects of the interaction between the piezoceramic bar vibration and the excitation device (a vacuum-tube generator of limited power), with characteristics of nonideal phenomenon and saturation phenomenon by intervals. In this case the following parameters were chosen: $\alpha_4=0.103\times Y$, $\alpha_5=-0.0604\times Y$, $\alpha_6=-0.12\times Y$, and $Y \in [0.1, 11.0]$.

For $\varepsilon > 0$ small, we may write Eq. (1) in the form

$$
\begin{aligned}
u_1' &= u_2 \\
u_2' &= -u_1 + \varepsilon(\alpha_1 u_2 + \alpha_2 u_2^2 - \alpha_3 u_2^3 - \alpha_4 u_3) \\
u_3' &= u_4 \\
u_4' &= -\alpha_0^2 u_3 + \varepsilon(\alpha_5 u_1 + \alpha_6 u_2 - \alpha_7 u_4)
\end{aligned}
\tag{7}
$$

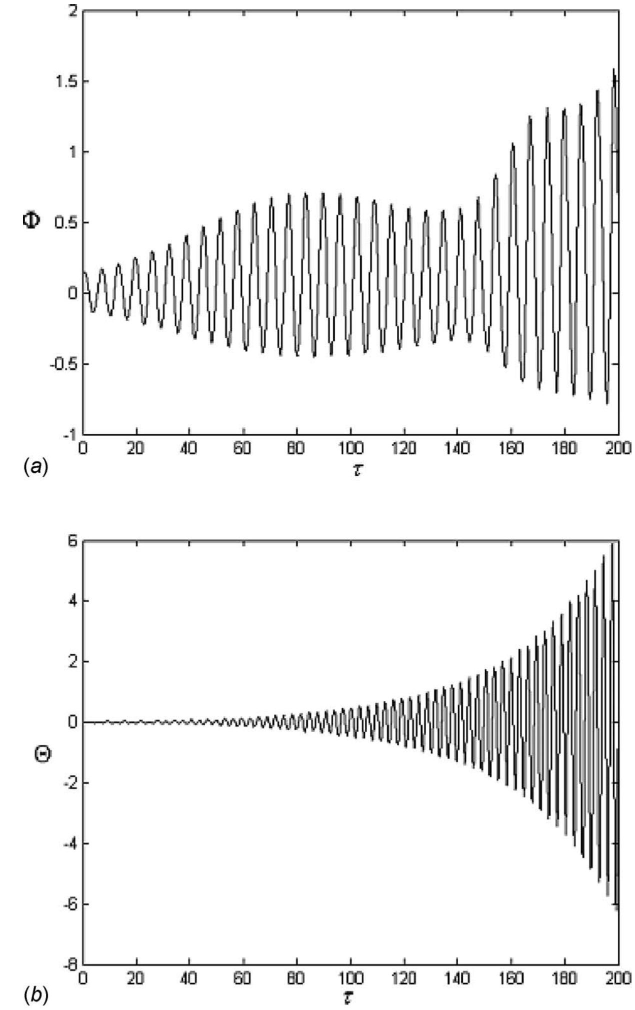

Fig. 8 (*a*) and (*b*) Response of instability when $\alpha_7=-0.06$

To determine the first approximated solution of Eq. (7), in accordance with the method of averaging of Palacios [5] and by considering $\alpha_0=\omega_1/\omega_0$, we will use the following change in variables:

$$u_1 = a_1 \cos(\tau+\beta_1), \quad u_3 = a_2 \cos(\alpha_0\tau+\beta_2) \tag{8}$$

$$u_2 = -a_1 \sin(\tau+\beta_1), \quad u_4 = -\alpha_0 a_2 \sin(\alpha_0\tau+\beta_2) \tag{9}$$

In Eq. (7), we obtain

$$
\begin{pmatrix} a_1' \\ a_1\beta_1' \\ a_2' \\ a_2\beta_2' \end{pmatrix} = \begin{pmatrix} -\varepsilon F_1 \sin(\tau+\beta_1) \\ -\varepsilon F_1 \cos(\tau+\beta_1) \\ -\varepsilon\alpha_0^{-1} F_2 \sin(\alpha_0\tau+\beta_2) \\ -\varepsilon\alpha_0^{-1} F_2 \cos(\alpha_0\tau+\beta_2) \end{pmatrix}
\tag{10}
$$

where

$$
\begin{aligned}
F_1 &= -\alpha_1 a_1 \sin(\tau+\beta_1) + \alpha_2 a_1^2 \sin^2(\tau+\beta_1) + \alpha_3 a_1^3 \sin^3(\tau+\beta_1) \\
&\quad - \alpha_4 a_2 \cos(\alpha_0\tau+\beta_2) \\
F_2 &= \alpha_5 a_1 \cos(\tau+\beta_1) - \alpha_6 a_1 \sin(\tau+\beta_1) + \alpha_7\alpha_0 a_2 \sin(\alpha_0\tau+\beta_2)
\end{aligned}
\tag{11}
$$

When $\alpha_0 \neq 1$, we determine a first approximation from the averaging equations of Eq. (10), considering a_1, a_2, β_1, and β_2 to be constants over one cycle and integrating (average) the equations over one cycle, the result is

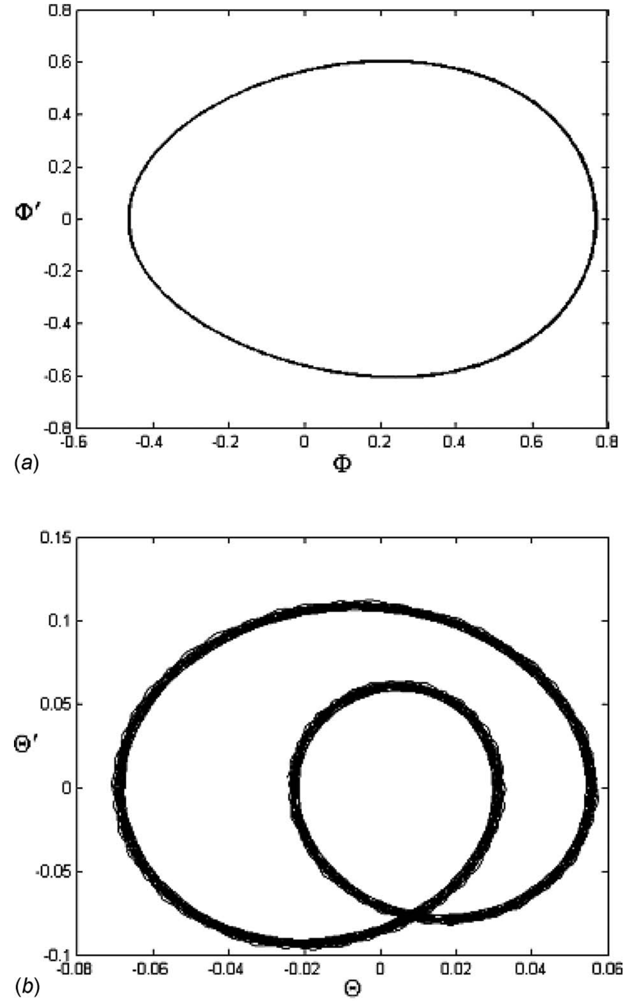

Fig. 9 (*a*) and (*b*) Response of periodic motion when $\alpha_7=0.0$

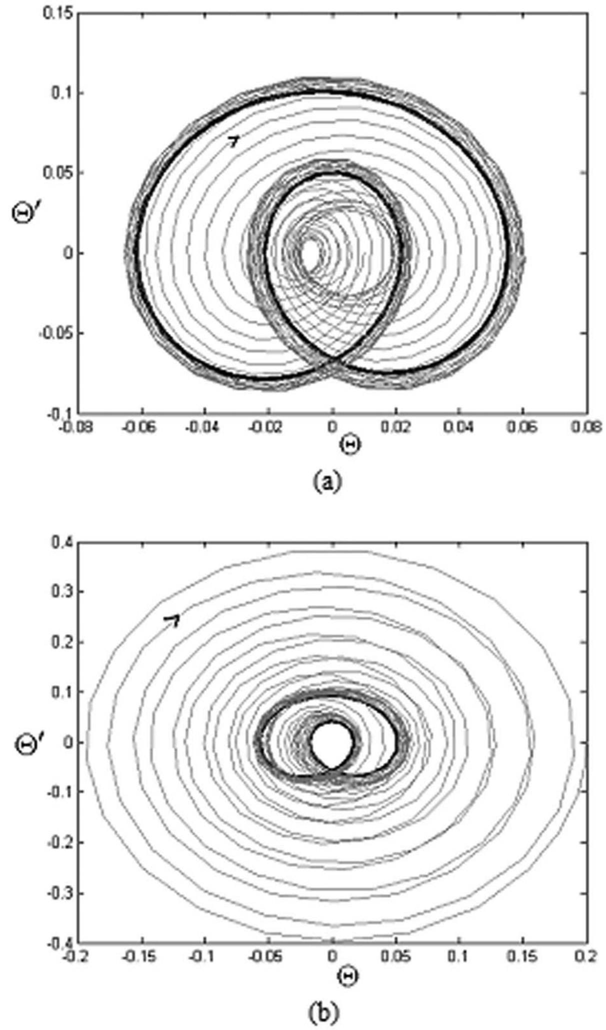

Fig. 10 Phase plane for initial disturbance: small (a) small and (b) large

$$a_1' = \varepsilon\left[\frac{\alpha_1}{2}a_1 - \frac{3\alpha_3}{8}a_1^3\right]$$

$$a_1\beta_1' = 0$$

$$a_2' = -\varepsilon\frac{\alpha_7}{2}a_2 \tag{12}$$

$$a_2\beta_2' = 0$$

Solving the four equations of Eq. (12), yields

$$a_1 = \frac{a_{01}}{\sqrt{\dfrac{3\alpha_3}{4\alpha_1}a_{01}^2 + \left(1 - \dfrac{3\alpha_3}{4\alpha_1}a_{01}^2\right)\exp(-\varepsilon\alpha_1\tau)}}, \quad \beta_1 = \beta_{01}$$

$$a_2 = a_{02}\exp\left(-\frac{\varepsilon\alpha_7}{2}\tau\right), \quad \beta_2 = \beta_{02} \tag{13}$$

Then, in the first approximation of the solutions described the nonlinear system dynamics of Eq. (1) are

$$\Phi = \frac{a_{01}}{\sqrt{\dfrac{3\alpha_3}{4\alpha_1}a_{01}^2 + \left(1 - \dfrac{3\alpha_3}{4\alpha_1}a_{01}^2\right)\exp(-\varepsilon\alpha_1\tau)}}\cos(\tau + \beta_{01})$$

$$\Theta = a_{02}\exp\left(-\frac{\varepsilon\alpha_7}{2}\tau\right)\cos(\alpha_0\tau + \beta_{02}) \tag{14}$$

where a_{01} and a_{02} are the initial disturbances, β_{01} and β_{02} are constants, and τ is the time dimensionless. The expression exp(.) denote the exponential function.

Next, we will considerer some comments on the response of the Eq. (1) through Eq. (14) with $X=1$.

(1) When $\alpha_0=2.0$, $\alpha_7=-0.06<0$ (contradicting the first equation of Eq. (4)), we have instability of the system $\Theta(0)=0.01$. This is confirmed by the expressions Φ and Θ in Eq. (14) and the numerical simulation of Eq. (1) that shows the amplitudes increasing to time long of the transducer-generator system in Fig. 8.
(2) When $\alpha_7=0.0$ and $\Theta(0)=0.1$ the response of Φ and Θ are of periodic motion permanent regime, as we observe in Fig. 9. Equation (14) shows $\Theta=a_{02}\cos(\alpha_0\tau+\beta_{02})$.
(3) When we consider the initial disturbances, large and small, Fig. 10 shows the numerical solutions of Eq. (1) for small

disturbance $\Theta(0)=0.01$ (Fig. 10(a)), where the amplitude increases until it reaches the limit cycle and large disturbance $\Theta(0)=0.2$ (Fig. 10(b)) where the amplitude decays until it reaches a limit cycle.

Conclusions

We presented, in this paper, some new numerical simulations and approximate analytical results on the nonlinear and nonideal behavior of a mathematical model of a piezoceramic transducer with limited energy supply. The ocurrence of the saturation phenomenon in this problem of a piezoceramic bar transducer powered by a vacuum-tube generated by a nonideal source, is a motivation for using saturation control in nonideal vibrating problem in Felix et al. [13].

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