INFLUENCE OF DELAY ON DYNAMICS OF NONIDEAL PENDULUM SYSTEMS

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In the majority of studies, the dynamics of various oscillatory dynamical systems are being conducted without taking into account the limitations of excitation source power. It is assumed that the power of excitation source considerably exceeds the power that consumes the vibrating system. In many cases, such idealization leads to qualitative and quantitative errors in describing dynamical regimes of such systems. Taking into account the limitation of excitation source power allows identify qualitative changes in dynamical characteristics and discover chaotic dynamical regimes (Krasnopol'skaya & Shvets, 1990; Krasnopol'skaya & Shvets, 1991; Shvets & Sirenko, 2012; Shvets & Makaseyev, 2012).

Another important factor that significantly affects the change of steady-state regimes of dynamical systems is the presence of different in their physical substance, factors of delay. The aim of this work is to study the influence of various factors of delay on steady-state regular and chaotic regimes of nonideal pendulum systems of the type "pendulum-electric motor". Mathematical model of the dynamical system "pendulum-electric motor" with taking into account various factors of delay can be written as the following system of equations (Shvets & Makaseyev, 2012; Shvets & Makaseyev, 2014):

$$\begin{cases} \dfrac{dy_1(\tau)}{d\tau} = Cy_1(\tau-\delta) - y_2(\tau)y_3(\tau-\gamma) - \dfrac{1}{8}\left(y_1^{\,2}(\tau)y_2(\tau) + y_2^{\,3}(\tau)\right), \\ \dfrac{dy_2(\tau)}{d\tau} = Cy_2(\tau-\delta) + y_1(\tau)y_3(\tau-\gamma) + \dfrac{1}{8}\left(y_1^{\,3}(\tau) + y_1(\tau)y_2^{\,2}(\tau)\right) + 1, \\ \dfrac{dy_3(\tau)}{d\tau} = Dy_2(\tau-\gamma) + Ey_3(\tau) + F. \end{cases} \quad (1)$$

where phase variables y_1, y_2 — describe the deviation of the pendulum from the vertical and phase variable y_3 — is proportional to the rotation speed of the motor shaft. The system parameters are defined by

$$C = -\delta_1 \varepsilon^{-2/3} \varpi_0^{\,-1},\ \ D = -\frac{2ml^2}{I},\ \ F = \frac{2l^{2/3}}{a^{2/3}}\left(\frac{N_0}{\varpi_0} + E\right), \quad (2)$$

where m — the pendulum mass, l — the reduced pendulum length, ϖ_0 — natural frequency of the pendulum, a — the length of the electric motor crank, $\varepsilon = \dfrac{a}{l}$, δ_1 — damping coefficient of the medium resistance force, I — the electric motor moment of inertia, E, N_0 — constants of the electric motor static characteristics.

Positive constant parameter γ was introduced to account the delay effects of electric motor impulse on the pendulum. We assume that the delay of the electric motor response to the impact of the pendulum inertia force is also equal to γ. The constant positive parameter δ characterizes the delay of the medium reaction on the dynamical state of the pendulum.

Let us consider two approaches that allow reducing the time-delay system (1) to the system of equations without delay. The first approach is as follows. Assuming a small delay, we can write

$$y_i(\tau-\gamma) = y_i(\tau) - \frac{y_i(\tau)}{d\tau}\gamma + ..., \quad i = 2, 3;$$

$$y_j(\tau-\delta) = y_j(\tau) - \frac{y_j(\tau)}{d\tau}\delta + ..., \quad j = 1, 2.$$

Then, if $C\delta \neq -1$, we get the following system of equations:

$$\begin{cases} \dfrac{dy_1}{d\tau} = \dfrac{1}{1+C\delta}\left(Cy_1 - y_2\left[y_3 - \gamma\left(Dy_2 + Ey_3 + F\right)\right]y_3 - \dfrac{1}{8}\left(y_1^2 y_2 + y_2^3\right)\right); \\ \dfrac{dy_2}{d\tau} = \dfrac{1}{1+C\delta}\left(Cy_2 + y_1 y_3 - y_1\gamma\left(Dy_2 + Ey_3 + F\right) + \dfrac{1}{8}\left(y_1^3 + y_1 y_2^2\right) + 1\right); \\ \dfrac{dy_3}{d\tau} = \left(1 - C\gamma\right)Dy_2 - \dfrac{D\gamma}{8}\left(y_1^3 + y_1 y_2^2 + 8y_1 y_2^2 + 8y_1 y_3 + 8\right) + Ey_3 + F. \end{cases} \qquad (3)$$

The obtained system of equations is already a system of ordinary differential equations. Delays are included in this system as additional parameters.

In order to approximate the system (1) another, more precise, method can be used. In the case $\gamma > 0$, $\delta > 0$ let us divide the segments $[-\gamma; 0]$ and $[-\delta; 0]$ into m equal parts. We introduce the following notation

$$y_1(\tau - \frac{i\delta}{m}) = y_{1i}(\tau),$$

$$y_2(\tau - \frac{i\gamma}{m}) = y_{2i}(\tau),$$

$$y_2(\tau - \frac{i\delta}{m}) = \tilde{y}_{2i}(\tau),$$

$$y_3(\tau - \frac{i\gamma}{m}) = y_{3i}(\tau), i = \overline{0, m}.$$

Then, using difference approximation of derivative we obtain

$$
\begin{cases}
\dfrac{dy_{10}(\tau)}{d\tau} = Cy_{1m}(\tau) - y_{20}(\tau)y_{3m}(\tau) - \dfrac{1}{8}\Big(y_{10}^2(\tau)y_{20}(\tau) + y_{20}^3(\tau)\Big); \\
\dfrac{dy_{20}(\tau)}{d\tau} = C\tilde{y}_{2m}(\tau) + y_{10}(\tau)y_{3m}(\tau) + \dfrac{1}{8}\Big(y_{10}^3(\tau) + y_{10}(\tau)y_{20}^2(\tau)\Big) + 1; \\
\dfrac{dy_{30}(\tau)}{d\tau} = Dy_{2m}(\tau) + Ey_{30}(\tau) + F; \\
\dfrac{dy_{1i}(\tau)}{d\tau} = \dfrac{m}{\delta}\Big(y_{1\,i-1}(\tau) - y_{1\,i}(\tau)\Big), \quad i = \overline{1,m}; \\
\dfrac{dy_{2i}(\tau)}{d\tau} = \dfrac{m}{\gamma}\Big(y_{2\,i-1}(\tau) - y_{2\,i}(\tau)\Big), \quad i = \overline{1,m}; \\
\dfrac{d\tilde{y}_{2i}(\tau)}{d\tau} = \dfrac{m}{\delta}\Big(\tilde{y}_{2\,i-1}(\tau) - \tilde{y}_{2\,i}(\tau)\Big), \quad i = \overline{1,m}; \\
\dfrac{dy_{3i}(\tau)}{d\tau} = \dfrac{m}{\gamma}\Big(y_{3\,i-1}(\tau) - y_{3\,i}(\tau)\Big), \quad i = \overline{1,m}.
\end{cases} \tag{4}
$$

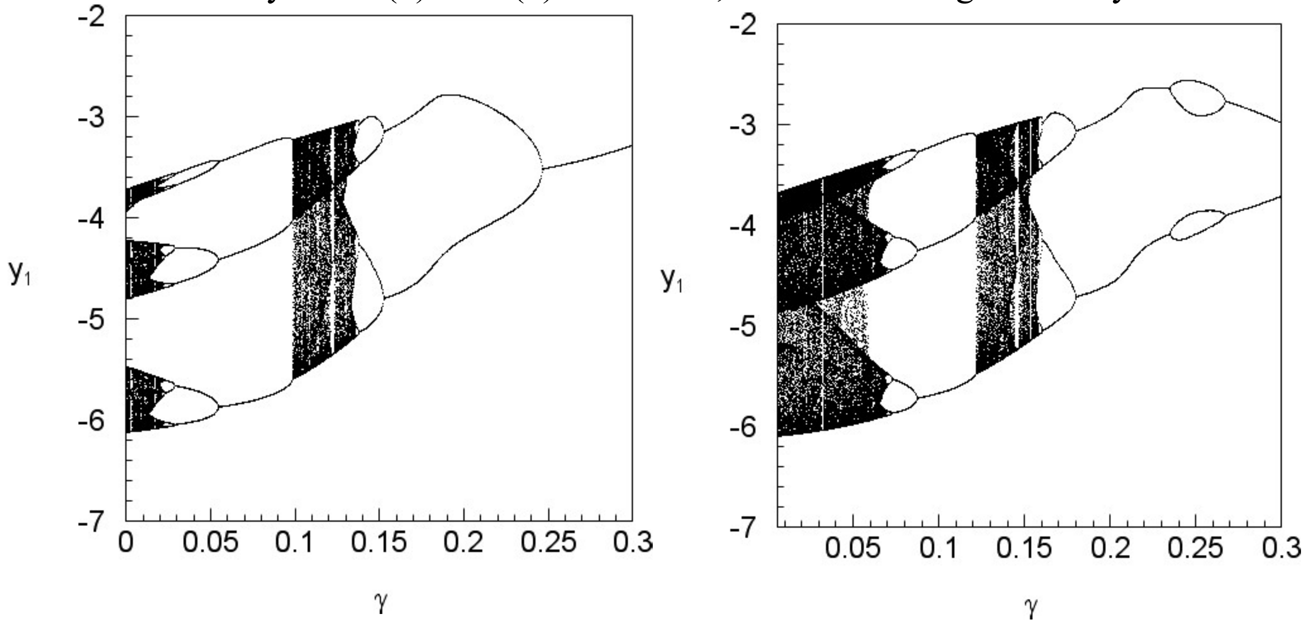
It is a system of ordinary differential equations of $(2m+3)$-th order. As in the system (3), the delays γ and δ are included in these systems as additional parameters. We note that the solutions y_1, y_2, y_3 of the system (1) are described by the functions y_{10}, y_{20}, y_{30} of the system (4).

Choosing a sufficiently large m in the system (4), the system (3) will be very well approximated by the system (4). The use of mathematical model (4) at $m = 3$, so the system (4) has 15 equations, is optimal for studying the influence of delay on regular and chaotic dynamics of "pendulum-electric motor" system (Shvets & Makaseyev, 2012; Shvets & Makaseyev, 2014).

Let us study the types of regular and chaotic attractors that exist in the systems (3) and (4). We consider the behavior of the systems (3) and (4) when parameters are $C = -0.1,\ D = -0.53,\ E = -0.6,\ F = 0.19$ and the delays $\delta = 0.15,\ 0 < \gamma \leq 0.3$. In fig. 1(a, b) phase-parametric characteristics (bifurcation trees) of three-dimensional system (3) and fifteen-dimensional system (4) are shown respectively. These figures illustrate the influence of the delay of interaction between pendulum and electric motor γ on chaotization of the system (1).

The areas of chaos in the bifurcation trees are densely filled with points. A careful examination of the obtained images allows not only to identify the origin of chaos in the systems, but also to describe the scenario of transition to chaos. So with a decrease of γ there are the transitions to chaos by Feigenbaum scenario (infinite cascade of period-doubling bifurcations of a limit cycle). Bifurcation points for the delay γ are the points of splitting the branches of the bifurcation tree (fig. 1(a, b)). In turn, the transition to chaos with an increase of the delay happens under the scenario of Pomeau — Manneville, in a single bifurcation, through intermittency.

A careful analysis of these figures allows to see qualitative similarity of characteristics of the systems (3) and (4). However, with increasing the delay the differences in the dynamics of these systems become very significant. For instance, when the delay of interaction between pendulum and electric motor $\gamma = 0.05$ the steady-state regime of three-dimensional system (3) is periodic and the attractor is limit cycle. Whereas at this values of the parameters and the delays fifteen-dimensional system (4) has steady-state chaotic dynamical regime.



(a) (b)

Fig. 1. Phase-parametric characteristics of the system (3) (a) and the system (4) (b)

It is also possible a different situation. For instance, at the delays $\gamma = 0.11$ the system (3) has chaotic steady-state regime of oscillations. Whereas at this values of the delay fifteen-dimensional system (4) has regular periodic dynamical regime and its attractor is limit cycle.

Conclusion. Taking into account various factors of delay in "pendulum-electric motor" systems is crucial. The presence of delay in such systems can affect the qualitative change in the dynamical behavior. It is shown, that in some cases the delay is the main reason of origination as well as vanishing of chaotic attractors. The use of three-dimensional mathematical model to study the dynamics of "pendulum — electric motor" systems is sufficient only at small values of the delay. For relatively high values of the delay multi-dimensional system of fifteen equations should be used.

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