NEW TYPE OF INTERMITTENCY AT CHAOTIC OSCILLATIONS OF HYDRODYNAMIC SYSTEMS WITH LIMITED EXCITATION

Shvets A.Yu.*, Krasnopolskaya T.S.**
*NTUU «Kyiv polytechnical institute», Pobedy ave., 37, 03057 Kyiv,
e-mail: alex.shvets@bigmir.net
** Institute of Hydromechanics, National Academy of Sciences of Ukraine
Zhelyabova Str., 8/4, 03680 Kyiv, e-mail: t.krasnopolskaya@tue.nl

Abstract

Oscillations of fluid free surface in a rigid tank raised by an electromotor of limited power-supply are considered. At examination of steady-stated chaotic regimes of oscillations of this deterministic system the new scenario of transition to chaotic motions is established and described. The described scenario is generalization of the known scenario of transition to chaos through intermittency in the sense of Pomeau--Manneville.

The problem of scenario disclosure of transition from one type steady-stated regimes to others, in particular, from the regular regimes to chaotic is one of the most interesting in theory of dynamic systems. To the present time enormous number of types of the strange attractors in dynamic systems of the most different nature are revealed and described. However, the number of known scenarios of transition between steady-stated regimes of different types remains rather small. Therefore, detection of new scenarios of transition to chaos is an interesting and actual scientific problem of nonlinear dynamics.

To the present time the three basic types of scenarios of transitions from the regular regimes to chaotic in the theory of dynamical systems are described, namely: (i) transition to chaos through the infinite cascade of bifurcations of period doubling of limit cycles (Feigenbaum scenario), (ii) transition to chaos through an intermittency in the sense of Pomeau - Manneville and (iii) transition to chaos through destruction of quasiperiodic attractors [1].

Let's consider a dynamical system which mathematical model in the following form:

$$
\begin{aligned}
\frac{dp_1}{d\tau} &= \alpha_1 p_1 - \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] q_1 + B\left(p_1 q_2 - p_2 q_1\right) p_2;\\
\frac{dq_1}{d\tau} &= \alpha_1 q_1 + \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] p_1 + B\left(p_1 q_2 - p_2 q_1\right) q_2 + 1;\\
\frac{d\beta}{d\tau} &= N_3 + N_1\beta - \mu_1 q_1; \qquad\qquad (1)\\
\frac{dp_2}{d\tau} &= \alpha_1 p_2 - \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] q_2 + B\left(p_1 q_2 - p_2 q_1\right) p_1;\\
\frac{dq_2}{d\tau} &= \alpha_1 q_2 + \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] p_2 + B\left(p_1 q_2 - p_2 q_1\right) q_1.
\end{aligned}
$$

The system of equations (1) for the first time has been obtained in works [2, 3] and describes oscillations of a fluid free surface in a rigid cylindrical tank raised by the electromotor of a limited power-supply. Here phase coordinates p_1, q_1 and p_2, q_2 - coefficients of amplitude expansions of fluid free surface oscillations of the first and second dominant modes; phase coordinate β - the detuning of the eigenfrequency of the dominant modes and a velocity of the shaft rotation of the electromotor; α_1 - reduced coefficient of a viscous damping force; μ_1 - coefficient of proportionality of the vibrational moment; N_1 - angle of an inclination of the static characteristic of the electromotor [4]. Parameters A, B are the constants depending on radius of a tank and height of filled fluid in it [5]. Value N_3 is determined by the formula [3]:

$$N_3 = \frac{1.417808 R^{\frac{2}{3}}}{a^{\frac{2}{3}} \omega_{11}} \left(N_0 - N_1 \omega_{11}\right), \qquad (2)$$

here R is the radius of a tank, ω_{11} - the eigenfrequency of the fluid free surface oscillation of the dominant modes, a is the length of a crank, N_0 is the constant component of the electromotor static characteristic.

The considered system is the deterministic dynamical system with limited excitation. Existence of chaotic regimes at limited excitation of a tank for the first time has been proved in the work [2]. However, in this work the proof of existence of such regimes is carried out only for a special case of planar oscillations of the fluid free surface. Occurrence of chaotic regimes in more general case of spatial oscillations of the free surface is established in the works [3, 6].

The system of equations (1) is essentially nonlinear one, therefore the determination of its exact solutions as analytical formulas in generally case is impossible. For determination of solutions of the system (1) numerical methods and algorithms were used. In work [6] the procedure of such numerical calculations is designed and in details described.

Let's assume that parameters of a system (1) are:

$$A = 1.112;\ B = 1.531;\ \alpha_1 = -0.1;\ \mu_1 = 0.5;\ N_3 = -0.1. \qquad (3)$$

At carrying-out of numerical calculations initial conditions were varied in a neighborhood of an origin of coordinates in the phase space of system of equations (1). As the carried out examinations have shown there are stable equilibrium states at $-0.1 < N_1 < -0.05$ in a system. At these states the coordinates have values:

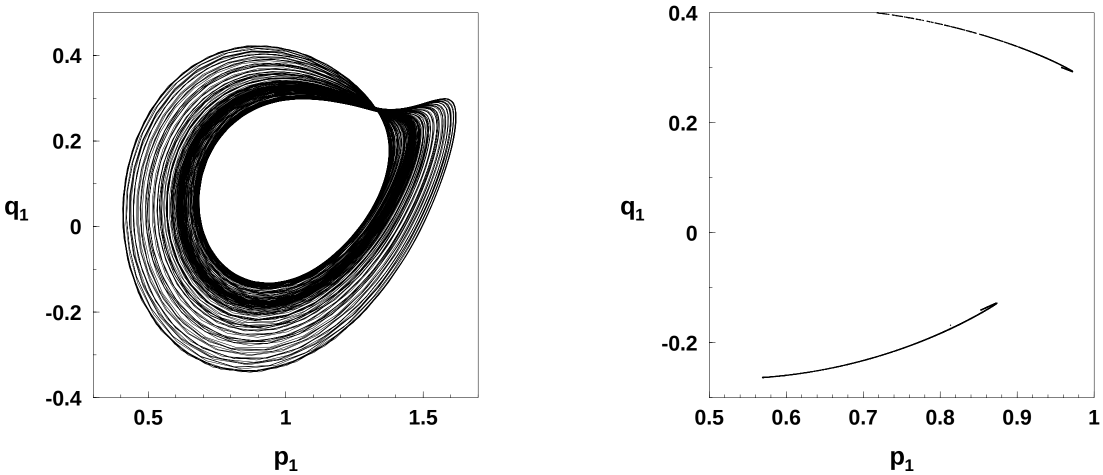
$$p_1 = const;\ q_1 = const;\ \beta = const;\ p_2 = 0;\ q_2 = 0.$$

Thus, all stable equilibrium states have zero coordinates for the second dominant mode in a neighborhood of an origin of coordinates in the phase space at $-0.1 < N_1 < -0.05$. At $N_1 = -0.1$ this equilibrium state loses its stability and in the system (1) a peculiar stable limit cycle with a zero second dominant mode arises as a result of the Andronov – Hopf bifurcation. The limit cycle has the following form:

$$p_1 = f_1(\tau);\ q_1 = f_2(\tau);\ \beta = f_3(\tau);\ p_2 = 0;\ q_2 = 0,$$

where $f_1(\tau)$, $f_2(\tau)$ and $f_3(\tau)$ are some periodic functions of τ.

At the value $N_1 = -0.10153$ a cascade of period doubling bifurcations of limit cycles starts in a system. This infinite cascade of period doubling bifurcations is completed by an origin of a chaotic attractor at $N_1 = -0.101632$. In fig. 1.a-b phase portrait projections of chaotic attractor constructed at the value $N_1 = -0.10164$ and its of Poincare section (by a plane $\beta = -1.55$) are shown, accordingly. The chaotic attractor has spiral structure, and its section of Poincare is quasiribbon chaotic point sets. Transition to chaos happens by Feigenbaum scenario [7]. We want to stress out the very interesting feature of the chaotic attractor when all bifurcations of a cascade of doubling of period of limit cycles and chaotic attractor have the zero second dominant mode of oscillations ($p_2 = q_2 = 0$). Such attractors we name as single-mode one.



a b

Fig.1. Projections of phase portrait (a) and of Poincare section (b) of chaotic attractor at $N_1 = -0.10164$.

At $N_1 = -0.10165$ the single-mode attractor disappears and a chaotic attractor of completely other type arises in the system. In fig. 2a-c different projections of the chaotic attractor which arises in the system at $N_1 = -0.10165$ are given. First of all it differs from the single-mode attractor by excitation of oscillations of the second dominant mode. Secondly, amplitudes of chaotic oscillations of the first dominant mode increase. Due to this the phase space volume, in which trajectories of the arisen chaotic attractor are localized, increases. So, in fig. 2a it is possible to see the small densely blacked out area in the neighborhood of the point (0,0). This blacked out area approximately corresponds to area of localization in the phase space of the missed single-mode attractor.

In fig. 2c the enlarged fragment of the chaotic attractor projection in the neighborhood of the point (0,0) is given. The study of this fragment allows to detect a noticeable similarity with the corresponding projection of the single-mode attractor (fig.1a). It makes clear that the mechanism of origin of the "double-mode" chaotic attractor is a result of an intermittency between the missed chaotic single-mode attractor and a saddle limit cycle existing on the neighborhood of localization of the single-mode attractor in the phase space. At $N_1 = -0.10165$ the single-mode attractor and the saddle cycle disappear and a new chaotic attractor arises in the system (1), motion along trajectories of which consist of three phases: laminar, turbulent and one more. The last one we name as coarse grained laminar phase. Motion which is close to periodic motion in the neighborhood of the missed limit cycle (see densely retraced trajectories at the left area in fig. 2a) corresponds to the laminar phase. At unpredictable beforehand moment of time turbulent splash happens and trajectories go away to the area of the missed single-mode chaotic attractor (densely blacked out area in the neighborhood of the point (0, 0) in fig. 2c). Then trajectories make chaotic wanderings along coils of the missed single-mode chaotic attractor during sufficiently long time. We named this phase of motions as coarse grained laminar as an analogy with the terminology used in the statistical physics [8]. Further, at unpredictable moment of time, there is a new turbulent splash and trajectories return to the area of the missed limit cycle. The above described process iterates an infinite number of times. Thus, the intermittency distinct from the Pomeau and Manneville classical types [9, 10] takes place.

In fig. 2d the projection of Poincare section (by a plane $\beta = -1.55$) of the double-mode chaotic attractor at $N_1 = -0.10165$ is given. As is apparent from figure the Poincare section loses the ribbon structure which existed in the section of the single-mode attractor and looks as some developed chaotic point set. However, the close investigation of fig. 2d allows to find, that the constituent of Poincare section of the double-mode chaotic attractor is the ribbon of the missed single-mode attractor. So, transition to chaos happens by the scenario which is generalization Pomeau - Manneville known scenarios [9, 10].

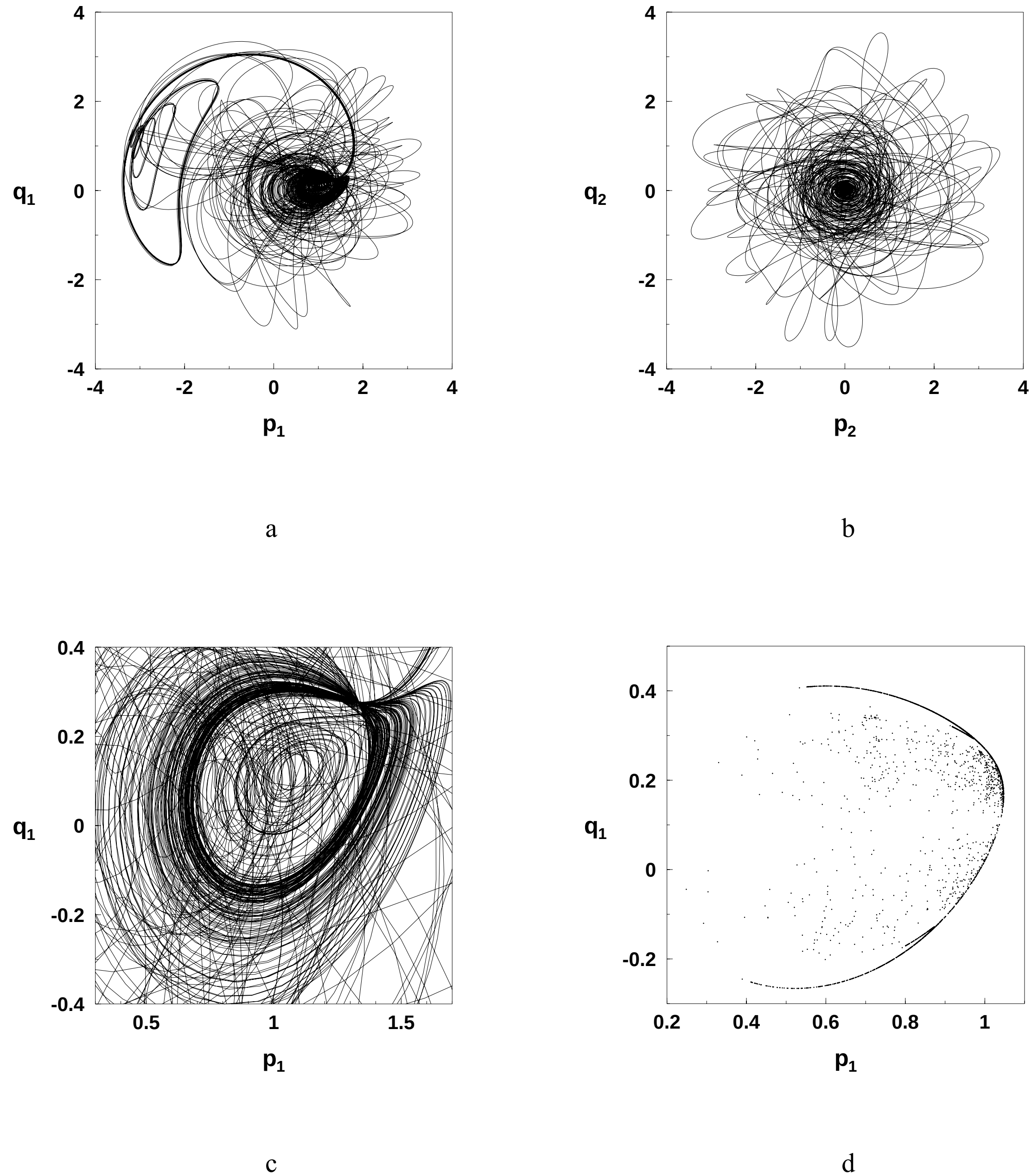
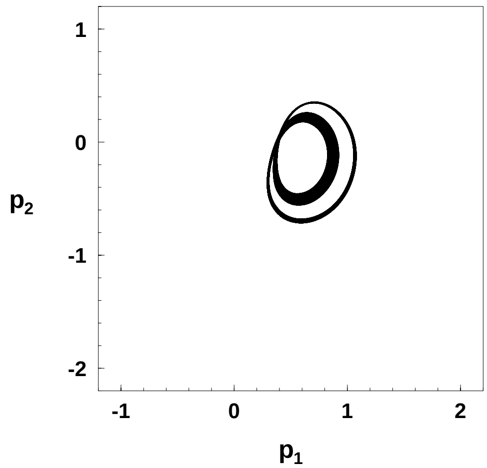
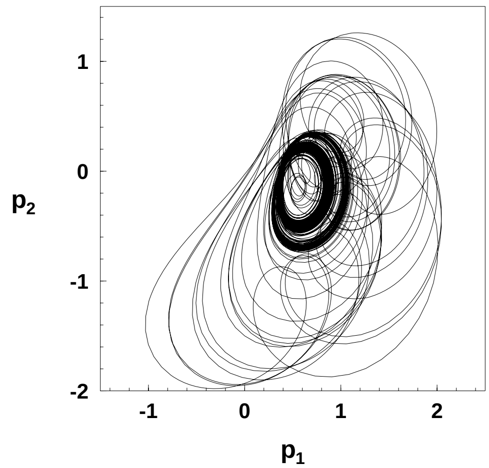


Fig.2. Projections of phase portrait (a-c) and of Poincare section (d) of chaotic attractor at $N_1 = -0.10165$.

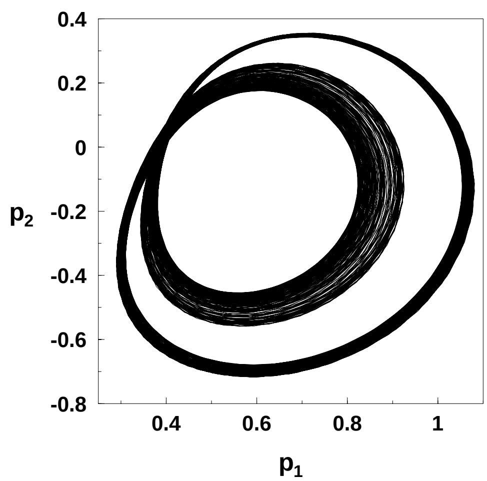
Let's consider now bifurcations which happen in the system (1) when the static characteristic of the electromotor is changing by the value of N_3. We shall suggest, that $N_1 = -1$, and values A, B, α_1 and μ_1 remain the same, as in (3). We shall study some features of transition from the regular regimes to chaotic at changing of the N_3 value. So at N_3=-0.38 there is a stable limit cycle in the system. At decreasing of N_3 values infinite cascade of period doubling bifurcations begins. This cascade brings to origination of a chaotic attractor at $N_3 \approx -0.395$. The arisen chaotic attractor exists in very small interval of N_3 changing and already at $N_3 \approx -0.39504$ is replaced by a chaotic attractor of other type as a result of an intermittency. The given situation reminds one which is considered earlier at study of origin of chaos at changing of the parameter N_1. However, in the latter case, one essential difference is present. Both limit cycles and a chaotic attractor originating by Feigenbaum scenario are not the single-mode ones. They have oscillations of both dominant modes.



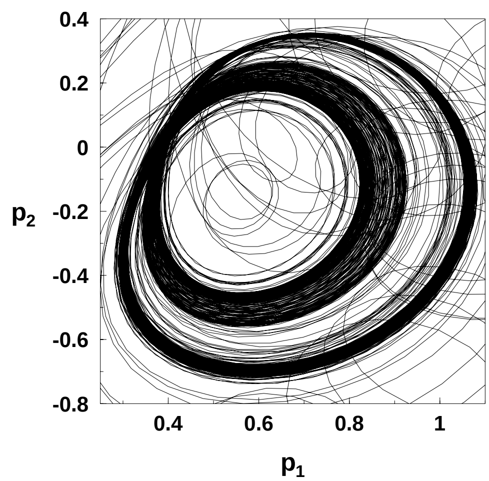
a



b



c

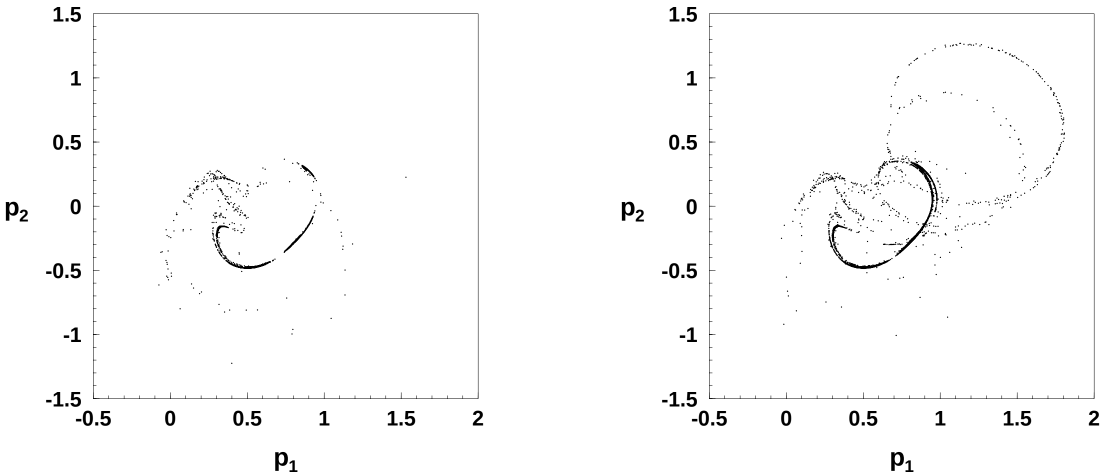


d

Fig.3. Projections of phase portraits of chaotic attractor at $N_3 = -0.39503$ (a, c); at $N_3 = -0.39504$ (b, d).

In fig. 3a-b projections of phase portraits of chaotic attractors constructed, accordingly, at $N_3 = -0.39503$ and $N_3 = -0.39504$ are given. The chaotic attractor presented in fig. 3b differs from the chaotic attractor presented in fig. 3a due to noticeable increasing of vibration amplitudes of both dominant modes. This gives essential increasing of the phase space volume in which the arisen attractor is localized. In fig. 3c-d these projections at large scale are presented. As it is well visible from these figures, the fragment of the projection of the chaotic attractor at $N_3 = -0.39504$ is qualitatively similar to the chaotic attractor at

$N_3 = -0.39503$. These figures make clear the mechanism of an intermittency of the origin of one attractor from another. In a point of a bifurcation the chaotic attractor from fig. 3c disappears and in the system (1) arise an attractor of new type, motions of trajectories of which consist of two phases. One of them, as well as earlier, we name coarse grained laminar, represent chaotic wanderings of trajectories along the arisen attractor in neighborhoods of trajectories of the missed attractor. At unpredictable moment of time trajectories "are broken" and leave to the remote areas of the phase space. It is the turbulent phase of motions of trajectories. Then trajectories again are returned in area of the missed attractor. This process infinite number of times iterates.



a b

Fig.4. Projections of Poincare section of chaotic attractors at $N_3 = -0.39503$ (a) and at $N_3 = -0.39504$ (b).

In fig. 4a-b Poincare sections (by a plane $\beta = -0.5$) of these attractors are given. Both Poincare sections are dot chaotic sets. One of sections (fig. 4b), as a fragment, contains a set qualitatively similar to the second section (fig. 4a), that confirms presence in the system of an intermittency such as "chaos - chaos". Transition to chaos by the scenario distinguished from classical scenarios of Pomeau - Manneville is observed here also.

References

[1] Kouznetsov, S.P. *Dynamic chaos*. Physmatlit, Moscow, 2001.
[2] Krasnopolskaya T.S., Shvets A.Yu. Regular and chaotic surface waves in a liquid in a cylindrical tank. *Priklanaya Mekhanika,* 26, №8, 85-93, 1990.
[3] Krasnopolskaya T.S., Shvets A.Yu. Chaotic surface waves in limited power-supply cylindrical tank vibrations. *Journal Fluids &Structures*, 8, №1, 1-18, 1994.
[4] Kononenko V.O. *Vibrating system with a limited power-supply*. Iliffe, London, 1969.
[5] Miles J.W. Internally resonant surface waves in circular cylinder. *J. Fluid Mech*., 149, 1-14, 1984.
[6] Shvets A.Yu. Scenarios of transitions "order - chaos" at resonance oscillations fluids in cylindrical tanks. *Proceedings of Institute of mathematics of NAS of Ukraine,* 3, №1, 216-249, 2007.
[7] Feigenbaum M.J. Quantative universality for a class of nonlinear transformations. *J. Stat. Phys*., 19, №1, 25-52, 1978.
[8] Gibbs J. W. *Elementary Principles in Statistical Mechanics*. Scribner, New Haven, 1902.
[9] Manneville P., Pomeau Y. Different ways to turbulence in dissipative dynamical systems. *Physica D.: Nonlinear Phenomena,* 1, №2, 219—226, 1980.
[10] Pomeau Y., Manneville P. Intermittent transition to turbulence in dissipative dynamical systems. *Comm. Math. Phys*., 74, №2,189-197, 1980.